A Convection-Decay Model for Simulating the Transmission of Flood Waves in Ephemeral Channels in Arid Zones

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Abstract: This paper presents a convection-decay model for simulating the transmission of flood waves in ephemeral channels in arid zones. The model is formulated and solved numerically by finite difference method on spreadsheet. The model has been applied on Yiba represented catchment in Saudi Arabia. The results are fairly good. Based on the proposed model, the average flood wave speed for the short reach is 10 km/hr and the average decay factor is 0.78 (hr⁻¹). For the long reach the average speed is 7.4 km/hr and the average decay factor is 0.26 (hr⁻¹). The results are fairly good. The model is capable of simulating flood wave movement in ephemeral streams.

Key words: Flood wave · Ephemeral Channels · Arid Zones · Flood Decay and Transmission losses

INTRODUCTION

Flood wave propagation in ephemeral streams is an important issue in the design of flood protection schemes. Several studies have focused on this issue, e.g. [1-8]. There are mainly two approaches to study flood wave propagation. The first approach is the flood routing technique that is based on hydrological aspects. The flood routing methods deals with the change of the shape of the hydrograph and its travel time down the channel. Among the many models used for flood routing, Kundzewics and Strupczewski [9] noted that the Muskingum method of flood routing has been extensively applied in river engineering practices since its introduction in the 1930s. Muskingum method is developed for perennial streams where the flow is continuous over the year round. However, in arid environment, most streams are ephemeral and catchments are often un-gauged. When performing flood routing in arid environment, the Muskingum parameters can be estimated using synthetic hydrograph methods [10] without using observed hydrograph because it is often unavailable. The inflow hydrograph could be estimated based on these synthetic methods and the outflow hydrographs at the end of the reach are generated with traditional Muskingum technique without incorporating transmission losses. O'Donnell [11, 12] developed the three-parameter Muskingum method to incorporate lateral inflows to a channel reach in the routing procedure of river channels. The approach is used to estimate the rate of lateral inflows and reconstruct hydrographs in temperate region catchments. Elfeki, et al. [13] have applied the three parameter Muskingum method with success. The other technique is the hydraulic routing where the hydraulic equations are involved. In this paper we proposes a kind of hydraulic routing approach with two parameters to describe the flood propagation in the ephemeral stream based on convection-decay type of model.

Fig. 1 illustrates the observed transmission of hydrographs over certain reaches in Yiba representative catchment in Saudi Arabia. The observed hydrograph transmission between two stations: Fig. 1a shows the flood wave transmission between station 422 and station 401 that are far apart about 10.5 km on 21 July 1985, while, Fig. 1b shows the translation of the flood wave between station 423 and 424 that are far apart about 33.2 km on 11 April 1985. The observed transmission losses during the
Fig. 1: Observed hydrograph transmission between two stations: (a) station 422 and 401 for 10.5 km channel length (21 July 1985), (b) station 423 and 424 for 33.2 km channel length (event 11 Apr 1985).

Fig. 2: WadiYiba catchment and its sub-basins projected on the satellite image of the region and the stations considered for the analysis (Red circles are rainfall stations and white circles with black dots are runoff stations).

Table 1: Some characteristics of the Runoff Stations at Yiba Catchment (Elfeki, et al., 2014)

<table>
<thead>
<tr>
<th>Station</th>
<th>Easting</th>
<th>Northing</th>
<th>Area (km²)</th>
<th>Length of the main stream (km)</th>
<th>Basin Slope (m/m)</th>
<th>Channel reach length (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>422</td>
<td>41°52'41.98&quot;</td>
<td>19°19'4.26&quot;</td>
<td>306.01</td>
<td>23.3</td>
<td>0.3322</td>
<td>10.5</td>
</tr>
<tr>
<td>401</td>
<td>41°48'42.35&quot;</td>
<td>19°17'2.15&quot;</td>
<td>764.96</td>
<td>35.07</td>
<td>0.2833</td>
<td>32.2</td>
</tr>
<tr>
<td>423</td>
<td>41°47'53.73&quot;</td>
<td>19°5'10.44&quot;</td>
<td>612.83</td>
<td>36.2</td>
<td>0.2657</td>
<td></td>
</tr>
<tr>
<td>424</td>
<td>41°37'58.89&quot;</td>
<td>19°0'45.53&quot;</td>
<td>2346.27</td>
<td>51.8</td>
<td>0.1969</td>
<td></td>
</tr>
</tbody>
</table>

translation of a flood waves are clear from these events. The shape of the hydrograph did not change dramatically when it moves downstream, yet both volume and peak are reduced.

Study Area: Yiba basin is located in Makkah Al-Mukaramah region in the Arabian Shield region of the country. It is in the southeastern part of Saudi Arabia. The boundaries of the basin area are bounded between 19°00’ and 19°30’ N latitude and 41°30’ and 42°00’ E longitudes. WadiYiba has an area of 2346 km² and drains its water towards the Red Sea (Fig. 2). The catchment of Yiba drains its water to the main channel and then to the Tehama coastal plain near the city of Habil. The alluvial fan of the wadi forms a suitable aquifer system for future development. Rainfall is very irregular and it increases from the coastal plain towards the mountains. It varies between 100 mm and 300 mm annually [14] where the climate is classified as arid. Stream channel network within the watershed are ephemeral. Most runoff events occur as a result of high intensity, short duration storms. Fig. 3 shows WadiYiba catchment and its sub-basins projected on the satellite image of the region and the stations considered for the analysis (Red circles are rainfall stations and white circles with black dots are runoff stations). Table 1 shows some characteristics of wadiYiba sub-basins.

Model Formulation: The mathematical model that describes the advection decay processes is expressed as,

$$\frac{\partial Q}{\partial t} + \frac{\partial Q}{\partial x} - \alpha Q = 0, \quad -\infty < x < \infty, \quad t > 0$$ (1)
Fig. 3: The conceptual model of the flood wave transmission in ephemeral stream.

Fig. 4: The numerical scheme used in Excel.

where,

\( v \) is the flood wave velocity,
\( Q \) is the discharge hydrograph,
\( \alpha \) is the decay coefficient and
\( x \) and \( t \) are the space and time variables respectively.

The analytical solution of such a model can be expressed by defining the initial condition of the model which is given by,

\[
Q(x, 0) = f(x) \text{ at } t = 0
\]  
(2)

where \( f(x) \) is the inflow hydrograph.

The solution in this case is given by,

\[
Q(x, t) = f(x - vt)e^{-\alpha t}
\]  
(3)

Since in the current case the inflow and outflow hydrographs are defined as numerical values and not functions, therefore a numerical model has been formulated using forward in time and backward in space finite difference scheme as follows,

\[
\frac{Q(x, t + \Delta t) - Q(x, t)}{\Delta t} = -v \left[ \frac{Q(x, t) - Q(x - \Delta x, t)}{\Delta x} \right] - \alpha Q(x - \Delta x, t)
\]

(4)

The model has been programmed on spreadsheet as shown in Fig. 5.

**Estimation of the Model Parameters:** Estimation of the model parameters from the data is made by the following equations:

The flood wave velocity is used estimated by

\[
v = \frac{L}{K}
\]  
(6)
Fig. 5: Spreadsheet model for performing the calculations.

Fig. 6: Estimation of the decay coefficient of the reach. With some manipulations, one obtains,

\[ L = \text{length of the reach between two stations} \]
\[ K = t_2 - t_1 \]

where \( L \) is the length of the reach between two stations and \( K \) is the travel time of the flood wave which is estimated by time lag between the starting points of the inflow and outflow hydrographs or the time between the two peaks of the hydrographs. For estimation of the decay coefficient, \( \alpha \), the following procedure is used,

\[ \frac{dQ}{dt} = -\alpha Q \]  

By integrating both sides one gets,

\[ \int_{Q_{1,\text{max}}}^{Q_{2,\text{max}}} \frac{Q_{1,\text{max}}}{Q_{1,\text{max}}} \, dQ = -\int_{t_1}^{t_2} \alpha \, dt \]

\[ \ln Q_{2,\text{max}} - \ln Q_{1,\text{max}} = -\alpha (t_2 - t_1) \]

\[ \alpha = \frac{\ln Q_{1,\text{max}} - \ln Q_{2,\text{max}}}{K} \]  

where \( K = t_2 - t_1 \)

Once, the parameters have been evaluated, the simulation can be performed and the comparison between the model results and the observed outflow can be achieved. Table 2 shows the estimated parameters of the model. The table shows that the average flood wave speed for the reach 422-401 is 10 km/hr and the average

<table>
<thead>
<tr>
<th>Reach</th>
<th>Date</th>
<th>( l ) (km)</th>
<th>( T_{lag} ) (hr)</th>
<th>( V ) (km/hr)</th>
<th>( Q_{in} ) (inflow)</th>
<th>( \ln Q_{in} )</th>
<th>( Q_{out} ) (outflow)</th>
<th>( \ln Q_{out} )</th>
<th>Decay factor (hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflow 422</td>
<td>1May1996</td>
<td>10.49</td>
<td>1.5</td>
<td>5.56525</td>
<td>111.78</td>
<td>4.72</td>
<td>57.52</td>
<td>4.65</td>
<td>0.416075</td>
</tr>
<tr>
<td>Outflow 401</td>
<td>12Jul1998</td>
<td>10.49</td>
<td>1</td>
<td>5.56525</td>
<td>26.67</td>
<td>3.26</td>
<td>17.35</td>
<td>2.85</td>
<td>0.426952</td>
</tr>
<tr>
<td>Inflow 5</td>
<td>22Apr1998</td>
<td>10.49</td>
<td>0.7</td>
<td>14.95871</td>
<td>144.76</td>
<td>4.98</td>
<td>52.76</td>
<td>3.97</td>
<td>1.4141974</td>
</tr>
<tr>
<td>Average</td>
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<td>10.49</td>
<td>1.15</td>
<td>10.03</td>
<td>106.41</td>
<td>4.49</td>
<td>44.60</td>
<td>3.70</td>
<td>0.786</td>
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<tr>
<td>Upper Limit</td>
<td></td>
<td>10.49</td>
<td>1.54</td>
<td>13.71</td>
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<td>5.52</td>
<td>83.26</td>
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<td>1.264</td>
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<td>0.76</td>
<td>9.34</td>
<td>33.29</td>
<td>3.68</td>
<td>29.54</td>
<td>3.14</td>
<td>0.303</td>
</tr>
<tr>
<td>Infow 423</td>
<td>20Aug1984</td>
<td>33.22</td>
<td>5.3</td>
<td>6.3</td>
<td>133.29</td>
<td>4.89</td>
<td>13.35</td>
<td>2.59</td>
<td>0.434121</td>
</tr>
<tr>
<td>Outflow 424</td>
<td>5Sep1985</td>
<td>33.22</td>
<td>3.3</td>
<td>11.1</td>
<td>182.30</td>
<td>6.21</td>
<td>20.29</td>
<td>3.33</td>
<td>0.412121</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>33.22</td>
<td>5.05</td>
<td>7.09</td>
<td>172.97</td>
<td>5.05</td>
<td>64.65</td>
<td>3.96</td>
<td>0.266</td>
</tr>
<tr>
<td>SD</td>
<td></td>
<td>0.00</td>
<td>2.00</td>
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<td>0.62</td>
<td>0.40</td>
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<tr>
<td>Upper Limit</td>
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<td>33.22</td>
<td>7.05</td>
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<tr>
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<td>4.53</td>
<td>93.13</td>
<td>4.03</td>
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<td>2.80</td>
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</tr>
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</table>
Fig. 7: Comparison between simulated and observed outflow hydrographs between station 422 and station 401 for different time events.

Fig. 8: Comparison between simulated and observed outflow hydrographs between station 423 and station 424 for different time events.

decay factor is 0.78 ($hr^{-1}$). For the reach 423-424 the average speed is 7.4 km/hr and the average decay factor is 0.26 ($hr^{-1}$).

RESULTS AND DISCUSSIONS

The proposed model has been applied on Yiba catchment. The results are presented in Fig. 7 and Fig. 8. Fig. 7 shows a comparison between the simulated and the observed outflow hydrographs between station 422 and station 401 for different time events. The figure shows fairly good agreement between the observed and simulated hydrographs. Fig. 8 shows a comparison between simulated and observed outflow hydrographs between station 423 and station 424 for different time events. The results are fairly good in terms of the peak flow and the shape of the hydrograph. However, in terms of the lag, there is an underestimation this may be due to the length of the reach which is relatively long (three times longer) in comparison with the other reach.
CONCLUSIONS

This paper presents a convection-decay model for simulating the transmission of flood waves in ephemeral channels in arid zones. The model is formulated and solved numerically by finite difference method on spreadsheet. The model has been applied on Yiba represented catchment in Saudi Arabia. Based on the proposed model, the average flood wave speed for the reach 422-401 is 10 km/hr and the average decay factor is 0.78 (hr^{-1}). For the reach 423-424 the average speed is 7.4 km/hr and the average decay factor is 0.26 (hr^{-1}). The results are fairly good. The model is capable of simulating flood wave movement in ephemeral streams.

REFERENCES